EXERCISE – I

SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. If z is a complex number such that |z| = 4 and $\arg(z) = \frac{5\pi}{6}$, then z is equal to

(A) $-2\sqrt{3} + 2i$ (B) $2\sqrt{3} + i$

(C) $2\sqrt{3} - 2i$ (D) $-\sqrt{3} + i$

2. The argument of the complex number

$\sin\frac{6\pi}{5} + i\left(1+\cos\frac{6\pi}{5}\right)$ is

(A) $\frac{6\pi}{5}$ (B) $\frac{5\pi}{6}$ (C) $\frac{9\pi}{10}$ (D) $\frac{2\pi}{5}$

3. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

(A) $z_1 + z_4 = z_2 + z_3$ (B) $z_1 + z_3 = z_2 + z_4$

(C) $z_1 + z_2 = z_3 + z_4$ (D) None of these

4. The curve represented by $Re(z)^2 = 4$ is

(A) a parabola (B) an ellipse

(C) a circle (D) a rectangular hyperbola

5. The inequality $|z - 4| < |z - 2|$ represents

(A) Re(z) > 0 (B) Re(z) < 0

(C) Re(z) > 2 (D) Re(z) > 3

6. The number of solutions of the system of equations $Re(z^2) = 0$, $|z| = 2$ is

(A) 4 (B) 3 (C) 2 (D) 1

7. If z ($\neq -1$) is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then |z| is equal to

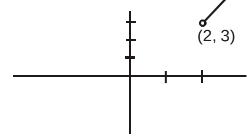
(A) 1 (B) 2 (C) 3 (D) 5

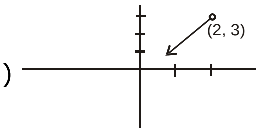
8. If $|z_1| = |z_2| = |z_3| = 1$ and z_1, z_2, z_3 are represented by the vertices of an equilateral triangle then

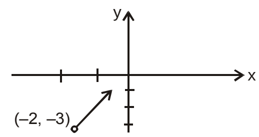
(A) $z_1 + z_2 + z_3 = 0$ (B) $z_1z_2z_3 = 1$

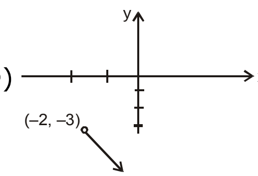
(C) $z_1z_2 + z_2z_3 + z_3z_1 = 0$ (D) None of these

9. If $\text{Arg}(z - 2 - 3i) = \frac{\pi}{4}$, then the locus of z is

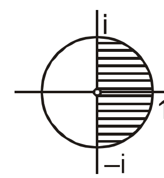
(A)


(B)


(C)


(D)


10. The locus of z which lies in shaded region is best represented by



(A) $|z| \le 1, \frac{-\pi}{2} \le \arg z \le \frac{\pi}{2}$

(B) $|z| = 1, \frac{-\pi}{2} \le \arg z \le 10$

(C) $|z| \ge 0, 0 \le \arg z \le \frac{\pi}{2}$ (D) $|z| \le 1, \frac{\pi}{2} \le \arg z \le \pi$

11. If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle |z| = 2 and if $z_1 = 1 + i\sqrt{3}$ then

(A) $z_2 = -2, z_3 = 1 + i\sqrt{3}$ (B) $z_2 = 2, z_3 = 1 - i\sqrt{3}$

(C) $z_2 = -2, z_3 = 1 - i\sqrt{3}$ (D) $z_2 = 1 - i\sqrt{3}, z_3 = -1 - i\sqrt{3}$

12. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)$ .... $\cos n\theta + i\sin n\theta) = 1$, then the value of θ is

(A) $4m\pi$, $m \in Z$ (B) $\frac{2m\pi}{n(n+1)}$, $m \in Z$

(C) $\frac{4m\pi}{n(n+1)}$, $m \in Z$ (D) $\frac{m\pi}{n(n+1)}$, $m \in Z$

13. If $x = a + b + c$, $y = a\alpha + b\beta + c$ and $z = a\beta + b\alpha + c$, where α and β are complex cube roots of unity then xyz equals

(A) $2(a^3 + b^3 + c^3)$ (B) $2(a^3 - b^3 - c^3)$

(C) $a^3 + b^3 + c^3 - 3abc$ (D) $a^3 - b^3 - c^3$

14. The equation $|z - 1|^2 + |z + 1|^2 = 2$ represents
(A) a circle of radius '1' (B) a straight line
(C) the ordered pair (0, 0) (D) None of these

15. The region of Argand diagram defined by
$|z - 1| + |z + 1| \le 4$ is
(A) interior of an ellipse (B) exterior of a circle
(C) interior and boundary of an ellipse
(D) None of these

16. Let z_1 and z_2 be to non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is
(A) 4 (B) 3 (C) 2 (D) $\sqrt{2}$

17. The curve represented by $|z| = \text{Re}(z) + 2$ is
(A) a straight line (B) a circle
(C) an ellipse (D) None of these

18. The set of values of $a \in R$ for which $x^2 + i(a - 1)x + 5 = 0$ will have a pair of conjugate imaginary roots is
(A) R (B) {1}
(C) $\{|a|\ a^2 - 2a + 21 > 0\}$ (D) None of these

19. If $z_1 = -3 + 5i$; $z_2 = -5 - 3i$ and z is a complex number lying on the line segment joining z_1 & z_2, then arg(z) can be
(A) $-\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

20. In G.P. the first term and common ratio are both $\frac{1}{2}(\sqrt{3} + i)$, then the absolute value of its n^{th} term is
(A) 1 (B) 2^n (C) 4^n (D) None of these

21. If $z = x + iy$ and $z^{1/3} = a - ib$ then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k equals
(A) 1 (B) 2 (C) 3 (D) 4

22. Let A, B, C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number
(A) $z_1 + z_2 - z_3$ (B) $z_2 + z_3 - z_1$
(C) $z_3 + z_1 - z_2$ (D) $z_1 + z_2 + z_3$

23. Find the least value of n ($n \in N$), for which $\left(\frac{1+i}{1-i}\right)^n$ is real
(A) 1 (B) 2 (C) 3 (D) 4

24. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to
(A) $\beta + i\alpha$ (B) $\alpha - i\beta$ (C) $\beta - i\alpha$ (D) $-\alpha - i\beta$

25. If $|z| = \max\{|z - 1|, |z + 1|\}$ then
(A) $|z + \bar{z}| = 1/2$ (B) $z + \bar{z} = 1$
(C) $|z + \bar{z}| = 1$ (D) None of these

26. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$
(A) is less than 6 (B) is more than 3
(C) is less than 12 (D) lies between 6 and 12

27. The vector $z = -4 + 5i$ is turned counter clockwise through an angle of 180º & stretched 1.5 times. The complex number corresponding to the newly obtained vector is
(A) $6 - \frac{15}{2}i$ (B) $-6 + \frac{15}{2}i$
(C) $6 + \frac{15}{2}i$ (D) None of these

28. Points z_1 & z_2 are adjacent vertices of a regular octagon. The vertex z_3 adjacent to z_2 ($z_3 \ne z_1$) is represented by
(A) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 + z_2)$ (B) $z_2 + \frac{1}{\sqrt{2}}(1 + i)(z_1 - z_2)$
(C) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_2 - z_1)$ (D) None of these

29. If z_1 & z_2 are two complex number & if $\arg\frac{z_1 + z_2}{z_1 - z_2} = \frac{\pi}{2}$ but $|z_1 + z_2| \ne |z_1 - z_2|$ then the figure formed by the points represented by 0, z_1, z_2 & $z_1 + z_2$ is
(A) a parallelogram but not a rectangle or a rhombus
(B) a rectangle but not a square
(C) a rhombus but not a square
(D) a square

30. The expression $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^n - \frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to

(A) zero (B) 2 sin n α

(C) 2 cos n α (D) None of these

31. If $p = a + b\omega + c\omega^2$; $q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where a, b, c $\neq$ 0 and ω is the complex cube root of unity then

(A) $p + q + r = a + b + c$ (B) $p^2+q^2+r^2=a^2+b^2+c^2$

(C) $p^2+q^2+r^2 =2(pq+qr+rp)$ (D) None of these

32. If $x^2 + x + 1 = 0$ then the numerical value of the expression

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\left(x^4+\frac{1}{x^4}\right)^2+\ldots\ldots$$

$$\ldots\ldots + \left(x^{27}+\frac{1}{x^{27}}\right)^2 \text{ is}$$

(A) 54 (B) 36 (C) 27 (D) 18

33. If α is non real and $\alpha = \sqrt[5]{1}$ then the value of $2^{|1+\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|}$ is equal to

(A) 4 (B) 2 (C) 1 (D) None of these

34. Number of roots of the equation $z^{10} - z^5 - 992 = 0$ with real part negative is

(A) 3 (B) 4 (C) 5 (D) 6

35. The points $z_1 = 3 + \sqrt{3}\, i$ and $z_2 = 2\sqrt{3} + 6i$ are given on a complex plane. The complex number lying on the bisector of the angle formed by the vectors z_1 and z_2 is

(A) $z = \frac{(3+2\sqrt{3})}{2} + \frac{\sqrt{3}+2}{2}i$ (B) $z = 5 + 5i$

(C) $z = -1 - i$ (D) None of these

36. The points of intersection of the two curves $|z - 3| = 2$ and $|z| = 2$ in an argand plane are

(A) $\frac{1}{2}(7 \pm i\sqrt{3})$ (B) $\frac{1}{2}(3 \pm i\sqrt{7})$ (C) $\frac{3}{2} \pm i\sqrt{\frac{7}{2}}$ (D) $\frac{7}{2} \pm i\sqrt{\frac{3}{2}}$

37. The equation of the radical axis of the two circles represented by the equations, $|z - 2| = 3$ and $|z - 2 - 3i| = 4$ on the complex plane is

(A) $3iz - 3i\bar{z} - 2 = 0$ (B) $3iz - 3i\bar{z} + 2 = 0$

(C) $iz - i\bar{z} + 1 = 0$ (D) $2iz - 2i\bar{z} + 3 = 0$

38. If $(1 + i)z = (1 - i)\bar{z}$ then z is

(A) $t(1 - i)$, $t \in R$ (B) $t(1 + i)$, $t \in R$

(C) $\frac{t}{1+i}$, $t \in R^+$ (D) None of these

39. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is

(A) 4 (B) 10 (C) 6 (D) 0

40. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{k\pi}{11}\right)$

(A) 1 (B) – 1 (C) – i (D) i

41. If the cube roots of unity are 1, ω, ω^2, then roots of the equation $(x - 1)^3 + 8 = 0$ are

(A) $-1, 1 + 2\omega, 1 + 2\omega^2$ (B) $-1, 1 - 2\omega, 1 - 2\omega^2$

(C) $-1, -1, -1$ (D) $-1, -1 + 2\omega, -1 - 2\omega^2$

42. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then arg z_1 – arg z_2 is equal to

(A) $-\frac{\pi}{2}$ (B) 0 (C) $-\pi$ (D) $\frac{\pi}{2}$

43. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

(A) a parabola (B) a straight line

(C) a circle (D) an ellipse

44. Let z, w be complex numbers such that $\vec{z} + i\vec{w} = 0$ and arg zw = π. Then arg z equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

45. If $|z^2 - 1| = |z^2| + 1$, then z lies on

(A) the real axis (B) the imaginary axis

(C) a circle (D) an ellipse

46. Let z_1 and z_2 be two roots of the equation ^{2} + az + b = 0, z being complex. Further, assume that he origin z_1 and z_2 form an equilateral triangle. Then
(A) $a^2 = b$ (B) $a^2 = 2b$ (C) $a^2 = 3b$ (D) $a^2 = 4b$

47. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$ then $\bar{z}\omega$ is equal to
(A) 1 (B) –1 (C) i (D) – i

48. If $z_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$, r = 1, 2,...... then $z_1 z_2 z_3$..... is equal to
(A) –1 (B) i (C) – i (D) 1

49. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to
(A) $\frac{1+\sqrt{2}}{2\sqrt{2}}$ (B) $\frac{1}{8}$ (C) $\cos\frac{\pi}{8}$ (D) $\frac{1}{2}$

50. The product of cube roots of –1 is equal to
(A) –2 (B) 0 (C) –1 (D) 4

51. If the complex numbers iz, z and z + iz represent the three vertices of a triangle then the area of the triangle is
(A) $\frac{1}{2}|z-1|$ (B) $|z|^2$ (C) $\frac{1}{2}|z|^2$ (D) $|z-1|^2$

52. Complex number z_1, z_2 and z_3 in AP
(A) lie on ellipse (B) lie on a parabola
(C) lie on line (D) lie on circle

53. If $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, \ldots\ldots C_n$ are constants and $C_n \neq 0$ then the value of n equals
(A) 2 (B) 4 (C) 6 (D) 8

54. If magnitude of a complex number 4 – 3i is tripled and is rotated by an angle π anticlockwise then resulting complex number would be
(A) –12+9i (B) 12 + 9i (C) 7 – 6i (D) 7 + 6i

55. If $|z - 2 - 3i| + |z + 2 - 6i| = 4$ where $i = \sqrt{-1}$ then locus of P(z) is
(A) an ellipse (B) ϕ
(C) segment joining the point 2 + 3i; – 2 + 6i
(D) None of these

56. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is
(A) 0 (B) 2 (C) 7 (D) 13

57. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $z_1+z_2+z_3|$ is
(A) equal to 1 (B) less than 1
(C) greater than 3 (D) equal to 3

58. If 1, α, α^2,......, α^{n-1} are nth roots of unity. The value of $(3 - \alpha)(3 - \alpha^2)(3 - \alpha^3)\ldots..(3 - \alpha^{n-1})$ is
(A) n (B) 0 (C) $\frac{3^n - 1}{2}$ (D) $\frac{3^n + 1}{2}$

59. In one root of the quadratic equation $(1 + i)x^2 - (7 + 3i)x + (6 + 8i) = 0$ is 4 – 3i, then the other root must be
(A) 1 + i (B) 4 + 3i (C) 1 – i (D) None of these

60. If P, P′ represent the complex number z_1 and its additive inverse respectively then the complex equation of the circle with PP′ as a diameter is
(A) $\frac{z}{z_1} = \overline{\left(\frac{z_1}{z}\right)}$ (B) $z\bar{z} + z_1\bar{z}_1 = 0$
(C) $z\bar{z} + \bar{z}z_1 = 0$ (D) None of these

61. If z = x + iy satisfies amp (z – 1) = amp (z + 3) then the value of (x – 1) : y is equal to
(A) 2 : 1 (B) 1 : 3 (C) –1 : 3 (D) does not exist

62. Let z ($\neq$2) be a complex number such that $\log_{1/2}|z - 2| > \log_{1/2}|z|$, then
(A) Re(z) > 1 (B) Im(z) > 1
(C) Re(z) = 1 (D) Im(z) = 1

63. The number of solutions of $z^3 + \bar{z} = 0$ is
(A) 2 (B) 3 (C) 4 (D) 5

64. If $iz^3 + z^2 - z + i = 0$, then $|z|$ equals
(A) 4 (B) 3 (C) 2 (D) 1

65. If $a > 0$ and the equation $|z - a^2| + |z - 2a| = 3$ represents an ellipse then a lies in
(A) (1, 3) (B) $(\sqrt{2}, \sqrt{3})$ (C) (0, 3) (D) $(1, \sqrt{3})$

66. If $w \neq 1$ is nth root of unity, then value of $\sum_{k=0}^{n-1} |z_1 + w^k z_2|^2$ is
(A) $n(|z_1|^2 + |z_2|^2)$ (B) $|z_1|^2 + |z_2|^2$
(C) $(|z_1| + |z_2|)^2$ (D) $n(|z_1| + |z_2|)^2$

67. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals
(A) 24 (B) 48 (C) 72 (D) 96

68. If z_1, z_2, z_3 are three complex numbers such that $4z_1 - 7z_2 + 3z_3 = 0$, then z_1, z_2, z_3 are
(A) vertices of a scalane triangle
(B) vertices of a right triangle
(C) points on a circle (D) collinear points

69. If $z = x + iy$ then the equation of a straight line $Ax + By + C = 0$ where $A, B, C \in R$, can be written on the complex plane in the form $\bar{a}z + a\bar{z} + 2C = 0$ where 'a' is equal to
(A) $\frac{(A+iB)}{2}$ (B) $\frac{A-iB}{2}$ (C) A+iB (D) None of these

70. If $z_1, z_2, z_3, \ldots, z_n$ lie on the circle $|z| = 2$, then the value of
$E = |z_1 + z_2 + \ldots\ldots + z_n| - 4\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots\ldots + \frac{1}{z_n}\right|$ is
(A) 0 (B) n (C) –n (D) None of these

71. The number of solutions of the equation in z, $z\bar{z} - (3 + i)z - (3 - i)\bar{z} - 6 = 0$ is
(A) 0 (B) 1 (C) 2 (D) infinite

72. If $1 + x^2 = \sqrt{3}\,x$ then $\sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)$ is equal to
(A) 48 (B) –48 (C) $\pm 48\,(\omega - \omega^2)$ (D) None of these

73. If $w\ (\neq 1)$ is a cube root of unity, then
$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$$ equals
(A) 0 (B) 1 (C) i (D) ω